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A NOTE ON THE HEAT TRANSFER CHARACTERISTICS OF CIRCULAR IMPINGING JET

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Abstract — For small separation distance between a nozzle and an impingement plate, the radial distribution of heat flux shows the existence of two secondary peaks around the stagnation point. As established in the literature, the outer peak occurs on account of transition in wall boundary layer flow. In this note, it has been shown, that the inner peak arises due to the non uniform turbulence in the jet.

NOMENCLATURE

- C_p, specific heat at constant pressure;
- d_e, nozzle diameter at exit section;
- f, boundary layer velocity function, dimensionless;
- F, G, functions defined in equations (5) and (6);
- K, thermal conductivity;
- Nu, Nusselt number, $\alpha d_e/K$;
- Pr, laminar Prandtl number, $\mu C_p/K$;
- Pr_t, turbulent Prandtl number, $\varepsilon_t/\varepsilon_k$;
- roz, cylindrical polar coordinate system fixed to plate;
- $r_1 o z_1$, cylindrical polar coordinate system fixed at nozzle exit;
- r_b, local jet radius;
- Re, Reynolds number, $\beta d_e^2/\nu$;
- t', fluctuating temperature, dimensionless;
- T, mean temperature;
- u', v', fluctuating components of velocities along r and z coordinates respectively;
- $\tilde{u}', \tilde{v}'$, r.m.s. values of u', v';
- U, V, mean velocity components in outer inviscid flow along r and z directions respectively;
- Z_n, separation distance between nozzle exit and the flat plate held normal to the flow;
- Z_∞, upstream influence of the plate extending back into the jet;

Greek symbols

- α, local heat-transfer coefficient;
- β, stagnation point velocity gradient;
- θ, non-dimensional temperature function $= \dfrac{T - T_w}{T_\infty - T_w}$;
- Ω, vorticity, $\partial v/\partial r - \partial u/\partial z$;
- ν, kinematic molecular viscosity;
- ε_k, eddy conductivity defined in equation (14);
- ε_t, eddy viscosity defined in equation (14);
- μ, molecular viscosity.

Subscripts

- 1, free jet;
- e, nozzle exit;
- w, wall;
- ∞, edge of wall boundary layer, edge of impingement region.

INTRODUCTION

THE FLOW field of a free jet impinging normally on a flat surface can be separated[1] into three regimes as shown in Fig. 1, i.e., the free jet flow, the impingement zone and the wall jet region. The impingement region is characterized by large pressure gradient, shear stress and heat-transfer rates.

For a circular air jet which is fully developed prior to impingement, ($Z_n > 8$–$10\,d_e$), the heat flux on the wall decreases monotonically from stagnation point. However, if the jet is not fully developed, the heat flux exhibits two well defined secondary peaks (Fig. 2), one located within the impingement region at $r = 0.5\,d_e$ (inner peak) and the other at about $r = 1.6\,d_e$–$2\,d_e$ (outer peak)[2, 3]. Gardon and Akfirat[2] have rightly attributed the outer peak to the transition of wall boundary layer flow from laminar to turbulent. The inner peak, according to them, is due to local thinning of the wall boundary layer. However such a thinning of wall boundary layer has not been observed either by Schrader in his analytical study[4] or Hrycak *et al.*[5] in their experiments. Thus there must be some other mechanism causing the inner peak. Surprisingly, even the recent work[6] on jet impingement has not taken note of the evidence against the explanation of Gardon and Akfirat for the inner peak.

The object of this note is to remove this anomaly by establishing that the inner peak occurs due to the non-uniform, mixing-induced turbulence in the developing jet. This result is based on a solution of incompressible

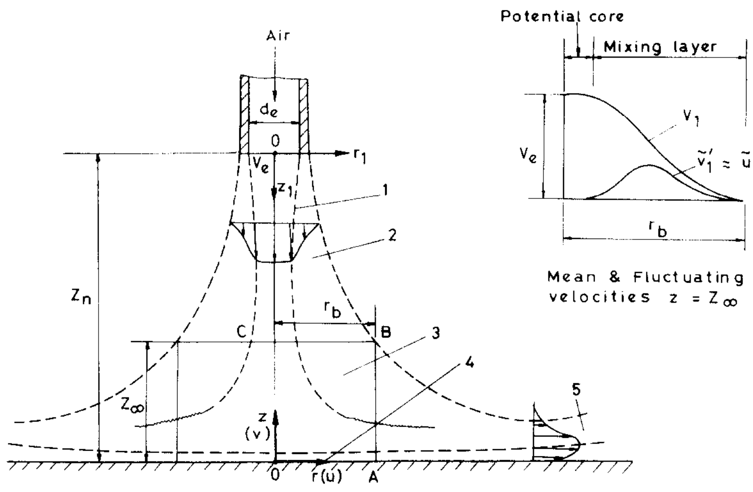

FIG. 1. Schematic diagram. 1: Potential core, 2: mixing layer, OABC: impingement region, 3: outer inviscid layer, 4: boundary layer, 5: wall jet region.

flow parabolic equations and a turbulence model for $Z_n/d_e = 2$ and 4 and is consistent with the predictions of Kestin *et al.*[7].

ANALYSIS

In Fig. 1 the geometry and boundary conditions of the problem are illustrated. We assume the jet to be at ambient temperature and the flat plate at a higher constant temperature. We assume that the deflection zone is confined in the radial direction to one free jet radius at $z = Z_\infty$[1]. The value Z_∞, which depends on Z_n/d_e, can be determined in terms of d_e from[1] and [4]. According to [2] we can sub-divide the flow into two parts: (a) wall boundary layer and (b) outer inviscid flow.

The exact equations governing the incompressible boundary layer flow are

$$\frac{\partial}{\partial r}(ur) + \frac{\partial}{\partial z}(vr) = 0 \tag{1}$$

$$u\frac{\partial \Omega}{\partial r} + v\frac{\partial \Omega}{\partial r} = \frac{\partial^2 \Omega}{\partial r^2} + \frac{\partial^2 \Omega}{\partial z^2} + \frac{1}{r}\left[\frac{\partial \Omega}{\partial r} + u\Omega - \frac{\Omega}{r}\right] + \frac{\partial^2}{\partial r \partial z}(\tilde{u}' - \tilde{v}') + \frac{1}{r}\left(\frac{\partial \tilde{u}'}{\partial z} - \frac{\partial \overline{u'v'}}{\partial r} + \frac{\overline{u'v'}}{r}\right) + \left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial r^2}\right)\overline{u'v'} \tag{2}$$

$$v\frac{\partial \theta}{\partial z} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial z^2} - \frac{\partial}{\partial z}(\overline{v't'}) \tag{3}$$

where

$$\Omega = \frac{\partial v}{\partial r} - \frac{\partial u}{\partial z}.$$

In the above equations both mean and turbulent velocities are non-dimensionalized with respect to $\sqrt{(\beta\nu)}$, the distances with respect to $\sqrt{(\beta/\nu)}$, where $\beta = (V_e/2Z_\infty)$ and the temperature

$$\theta = \frac{T - T_w}{T_\infty - T_w}. \tag{4}$$

At $z = Z_\infty$ we assume

$$V = -V_1 = -V_e F'(r_1) \tag{5}$$

where the function $F'(r_1)$ can be chosen from[8]. For the outer inviscid flow, assuming that V varies linearly with distance from the wall and by matching at $z = Z_\infty$, and noting that $r = r_1$, we can deduce that

$$U = 2G(r) \quad \text{and} \quad V = -2F'(r)z, \quad 0 \le r \le r_b \tag{6}$$

where

$$G(r) = F(r) - \frac{1}{r}\int F(r)\mathrm{d}r.$$

For viscous flow we can then assume

$$u = 2G(r)f'(z), \quad v = -2F'(r)f(z) \tag{7}$$

Turbulence model

Wolfstein[9] has developed a sophisticated turbulence model for a plane, fully developed, impinging jet where he solved the turbulent kinetic energy equation by finite difference technique employing the Kolmogorov–Prandtl hypothesis. For application to this problem where the impinging jet is axisymmetrical and still developing, the Wolfstein model needs some restructuring, particularly because the above anomaly in heat flux does not exist for fully developed jets. Although such an approach can, no doubt, give better results, it involves considerable numerical complexities. On the other hand, since the aim of this note is to explore the influence of non-uniform jet turbulence on heat flux near the stagnation point, we attempt to develop a simpler model based on the reported measurements.

At $z = Z_\infty$, from [10] and [11] we can assume

$$\tilde{u}_1' = 0.026(Z_n - Z_\infty)\left|\frac{\partial V_1}{\partial r_1}\right| \tag{8}$$

where a tilde over the symbol represents the r.m.s. value. According to [8], we can assume $\tilde{u}_1' = \tilde{v}_1'$.

Belov *et al.*[12] have measured for circular impinging jet ($2 \le Z_n/d_e \le 5$) the $\tilde{u}'$ component across the boundary layer close to the stagnation point and correlated their results by the following expression

$$\tilde{u}' = \frac{u}{U_\infty}\tilde{u}'_\infty \qquad (9)$$

where the subscript ∞ refers to the edge of wall boundary layer.

Boldman *et al.*[13] carried out extensive surveys of streamwise turbulence in the r, z plane of impingement region ($Z_n = 7d_e$). They found that the turbulence intensity is almost constant in the vertical direction from the plate up to $z = Z_\infty$, which means

$$\tilde{u}'_\infty = \tilde{v}'_1 = 0.026(Z_n - Z_\infty)\, V_e F''(r_1). \qquad (10)$$

However we have not come across any such measurements of $\tilde{v}'$. Therefore, for simplicity, we assume

$$\tilde{v}' = \tilde{u}'. \qquad (11)$$

Further, we assume

$$-\overline{u'v'} = 0.15(\tilde{u}' + \tilde{v}'), \qquad (12)$$

so that eddy viscosity

$$\varepsilon_t = \frac{-\overline{u'v'}}{\partial u/\partial z}. \qquad (13)$$

The eddy heat transfer term $-\overline{v't'}$ can be evaluated by the concept of a turbulent Prandtl number

$$Pr_t = \frac{\varepsilon_t}{\varepsilon_k} \quad \text{where} \quad \varepsilon_k = \frac{-\overline{v't'}}{\partial\theta/\partial z}. \qquad (14)$$

Assuming $Pr = Pr_t = 1$, we find $\varepsilon_k = \varepsilon_t$. The problem of modelling Reynolds stress in the outer portion of the boundary layer is complicated by the strong interaction between the outer jet-like flow and the inner flow near the wall with strong vorticity and pressure gradient. Therefore we use equation (12), only in the inner portion of the boundary layer. Towards the edge, ($z \to \infty$), we assume that the eddy viscosity (ε_t) tends to that in the free jet. In this way we develop an exponential function for ε_t which follows in the inner part, equation (12), and at the boundary layer edge matches with that in the free jet. The computations are thus of iterative nature.

By substituting the required quantities in (2) and (3) we get the following differential equations:

$$f^{IV} + B_1(f'f''' + f''^2) + B_2 ff''' + f''(B_3 f' + B_4) + B_5 f'^2 + B_6 f^2 + B_7 f = 0 \qquad (15)$$

$$\theta'' + B_8\theta' = 0 \qquad (16)$$

where $B_1, B_2, \ldots, B_8$ are the terms involving functions G, F', F, etc., and B_1, B_6 and B_8 contain mainly turbulence terms. These coefficients have different values in inner and outer portions of boundary layer and are derived as explained earlier.

Boundary conditions

$$f(0) = f'(0) = \theta(0) = 0$$
$$f'(\infty) = 1, \quad f''(\infty) = 0, \quad \theta(\infty) = 1. \qquad (17)$$

Equations (15) and (16) subject to (17) were solved numerically using fourth order Runge–Kutta algorithm by employing the concept of local similarity, which according to Eckert[14] gives satisfactory results in the neighbourhood of stagnation point.

RESULTS AND DISCUSSIONS

We have

$$\frac{Nu}{\sqrt{Re}} = \theta' \quad (\text{at } z = 0) \qquad (18)$$

where

$$Nu = \frac{\alpha d_e}{K} \quad \text{and} \quad Re = \frac{\beta d_e^2}{\nu}. \qquad (19)$$

The calculations for $Z_n/d_e = 2, 4$ and $Re = 11{,}000$ are presented in Fig. 2. A formal assumption of a value for Re (i.e. V_e, d_e and ν) becomes necessary because r_b appears explicitly in functions F and G and needs to be non-dimensionalised using β and ν. However, the heat flux in terms of $Nu/\sqrt{Re}$ is virtually uninfluenced by the input data since the coefficients $B_1, B_2, \ldots, B_8$ are weak functions of input numerical data. This was confirmed by computations by choosing different numerical values for V_e, d_e and ν.

From Fig. 2 it is clear that when turbulence is considered the heat flux exhibits a peak value, hitherto referred to as the inner peak. The present calculation closely follows the trend of measurements[2, 3, 6]. The mass-transfer data of [6] are converted to heat flux using the analogy between heat and mass transfer. The quantitative difference between these results could be due to several simplifying assumptions in the formulation.

The measurements of Gardon and Cobonpue[3] (using hollow cylindrical tubes fixed to an air supply line for producing jets) show that the inner peak is present for $Z_n/d_e = 2$ and 4, whereas the mass-transfer coefficients recorded by Koopman and Sparrow[6] (using sharp edged jet orifices) exhibit the inner peak only at $Z_n = 2d_e$. For higher nozzle to plate separations the inner peak disappears. This apparent discrepancy in the occurrence of the inner peak and its disappearance for higher values of Z_n/d_e can be explained by examining the accompanying changes in the turbulence structure of the jet.

The turbulence intensity in the potential core, and particularly on the jet axis, which is negligible at the exit starts rising as Z_1/d_e increases, resulting in progressively higher values of stagnation point heat-transfer rates. This trend continues for 6–8 dia downstream for Gardon and Cobonpue and 4 dia for Koopman and Sparrow. Once the potential core is engulfed by the entrainment process, the mean velocity

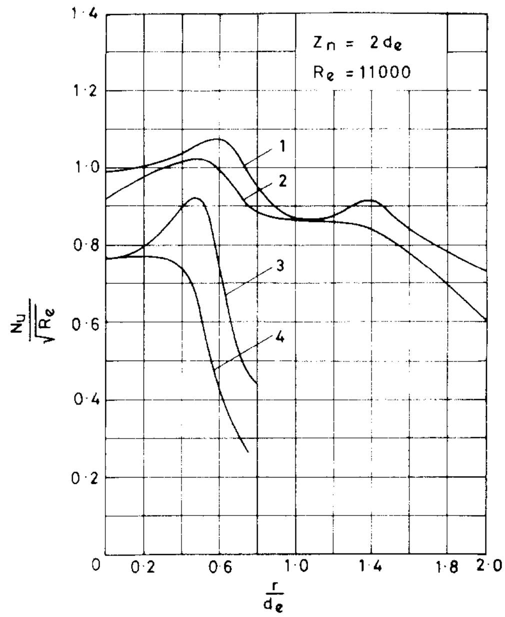

FIG. 2(a). Heat flux distribution, 1: $Z_n = 2d_e$, $Re = 11{,}000$. Gardon and Akfirat[2], 2: Koopman and Sparrow[6], 3: present result (with turbulence), 4: present result (mean flow only).

starts falling off, bringing about a decrease in the turbulence intensity on the axis, consequently the stagnation point heat flux starts falling off. During this process the radial distribution of turbulence intensity undergoes rapid changes with the peak value which hitherto was located at the centre line of the mixing region, $r = 0.5\,d_e$, starts shifting towards the axis[2]. As a consequence of this, the inner peak moves towards the stagnation point with the difference between the two heat fluxes becoming progressively smaller and finally vanishing. This happens at $Z_n/d_e = 6$ for Gardon and Cobonpue and at $Z_n = 4\,d_e$ for Koopman and Sparrow. The outer peak also starts becoming less distinct because the heat-transfer rates in the stagnation point region by now will have attained such higher values that the scope for their further rise as a consequence of the transition from the laminar to turbulent boundary layer flow is correspondingly diminished[2].

The semi-empirical turbulence model developed in this work cannot be extended to predict the disappearance of the inner peak because of its limited validity, restricted to the mixing region surrounding the potential core. Extrapolation of this model to higher values of Z_n/d_e is improper if we consider the following argument.

According to Townsend[15], in free mixing layers, such as the mixing region around the potential core of the circular free jet, there exists a near similarity and self preservation of mean velocity, turbulence intensities and turbulent shear. This phase of the flow exists only up to the apex of the potential core. The measurements of Davies *et al.*[10] confirm this hypothesis and expression (8) is developed on this basis. Beyond the apex of the potential core, this near similarity breaks down.

CONCLUSION

Based on a solution of incompressible turbulent flow equations, incorporating a semi-empirical turbulence model, we have established that the mechanism leading to the occurrence of the inner peak is the strong influence of mixing-induced, non uniform turbulence in the developing jet.

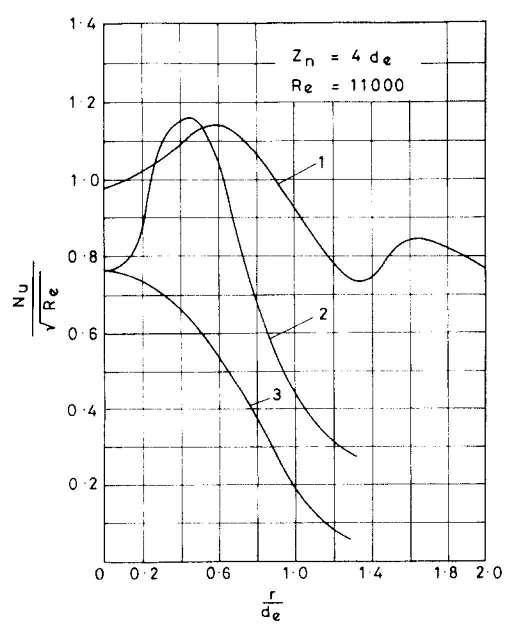

FIG. 2(b). Heat flux distribution, 1: $Z_n = 4d_e$, $Re = 11{,}000$. Gardon and Cobonpue[3], 2: present result (with turbulence), 3: present result (mean flow only).

REFERENCES

1. I. Tani and Y. Komatsu, Impingement of round jet on flat plate, *Proc. XI Int. Conf. Appl. Mech.* pp. 672–676. Springer-Verlag (1966).
2. R. Gardon and J. Akfirat, The role of turbulence in determining the heat-transfer characteristics of impinging jets, *Int. J. Heat Mass Transfer* **8**, 1261–1272 (1965).
3. R. Gardon and J. Cobonpue, Heat transfer between a flat plate and jets of air impinging on it, *1961 Int. Development in Heat Transfer*, Part 2, pp. 454–460, ASME, New York (1961).
4. H. Schrader, Trocknung feuchter Oberflächen mittels Warmluftstrahlen — Strömungsvorgang und Stoffübertragung, Forschungeh, Ver-Dtsch. Int. **27**, 484 (1961), Beilage zu Forschung aufdem Gebiete des Ingenieurwesens, **27B**.
5. P. Hrycak, D. T. Lee, J. W. Gauntner and J. N. B. Livingwood, Experimental flow characteristics of a single turbulent jet impinging on a flat plate, NASA TN D-5690 (1970).
6. R. N. Koopman and E. M. Sparrow, Local and average transfer coefficients due to an impinging row of jets, *Int. J. Heat Mass Transfer* **19**, 673–687 (1976).
7. J. Kestin, P. F. Maeder and H. E. Wang, Influence of turbulence on the transfer of heat from plates with and without a pressure gradient, *Int. J. Heat Mass Transfer* **3**, 133–154 (1961).
8. G. N. Abramovitch, *Theory of Turbulent Jets*, M.I.T. Press, Cambridge, MA (1963).

9. M. Wolfstein, Some solutions of the plane turbulent impinging jet, *J. Bas. Engng.* **92D,** 915–920 (1970).
10. P. O. A. L. Davies, M. J. Fisher and M. J. Barratt, The characteristics of turbulence in the mixing region of a round jet, *J. Fluid Mech.* **15**(3), 337–367 (1963).
11. J. C. Lawrence, Intensity, scale and spectra of turbulence in mixing region of free subsonic jet, NACA Rep. 1292 (1956).
12. I. A. Belov, G. P. Gorshikov, B. C. Komarov and B. C. Terpegoriev, Experimental study of gas dynamic parameters in the jet flow impinging upon a normal flat plate, *Izv. Akad. Nauk S.S.R., Mekh. Zhidk. i. Gaza* **2,** 139–142 (1971).
13. D. R. Boldman and P. F. Brinich, Mean velocity, turbulence intensity and scale in a subsonic turbulent jet impinging normal to a large flat plate, NASA Tech. Paper 1037 (1977).
14. E. R. G. Eckert, Research during the last decade on forced convection heat transfer, *1961 Int. Development in Heat Transfer,* Part 2, pp. ii–vi, ASME, New York (1961).
15. A. A. Townsend, Structure of turbulent shear flow, Cambridge University Press (1956).

NOTE SUR LES CARACTERISTIQUES DU TRANSFERT THERMIQUE POUR DES JETS INCIDENTS CIRCULAIRES

Résumé—Pour une petite distance entre l'orifice et la plaque, la distribution radiale du flux thermique montre l'existence de deux pics secondaires autour du point d'arrêt. Comme il est déjà établi, le pic extérieur rend compte de la transition de la couche limite en paroi. Dans cette note, on montre que le pic intérieur est la cause de la non uniformité de la turbulence dans le jet.

BEMERKUNG ZUR CHARAKTERISTIK DES WÄRMEÜBERGANGS AM STAUPUNKT VON KREISFÖRMIGEN STRAHLEN

Zusammenfassung—Bei kleiner Entfernung zwischen Düse und Aufprallplatte zeigt die radiale Verteilung des Wärmestroms zwei Sekundär-Maxima um den Stagnationspunkt. Es ist aus der Literatur bekannt, daß das äußere Maximum infolge eines Umschlags der Grenzschichtströmung an der Wand zustandekommt. In der vorliegenden Arbeit wurde gezeigt, daß das innere Maximum durch die ungleichförmige Turbulenz im Strahl entsteht.

О ТЕПЛООБМЕННЫХ ХАРАКТЕРИСТИКАХ КРУГЛЫХ НАТЕКАЮЩИХ СТРУЙ

Аннотация — При небольшом расстоянии между соплом и пластиной наблюдалось наличие двух вторичных пиков вокруг точки торможения в радиальном распределении плотности теплового потока. Отмечавшийся в других работах внешний пик обязан переходу потока в погранслойное течение на стенке. В данной работе показано, что внутренний пик возникает в силу неоднородности турбулентности в струе.